Nearly Tight Bounds for Robust Proper Learning of Halfspaces with a Margin

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UC San Diego

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Google

Agnostic Proper Learning of Halfspaces

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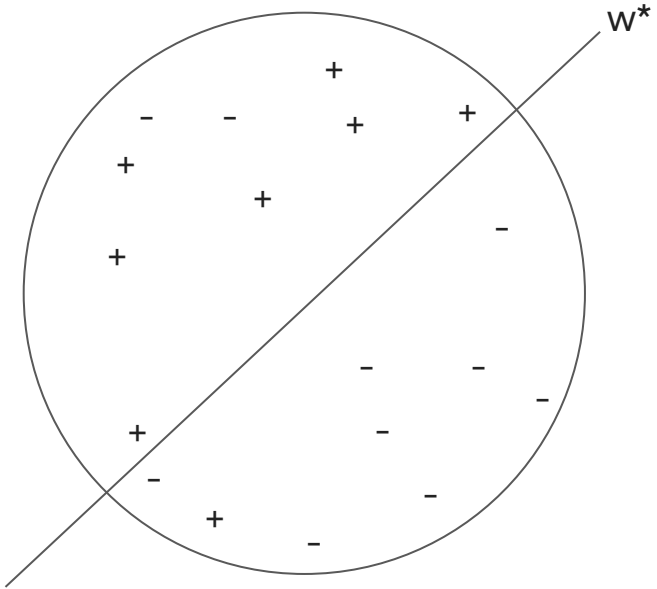

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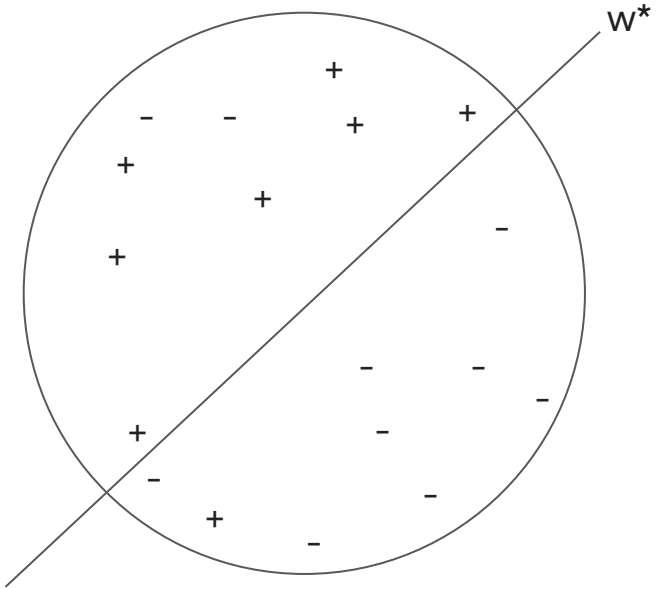

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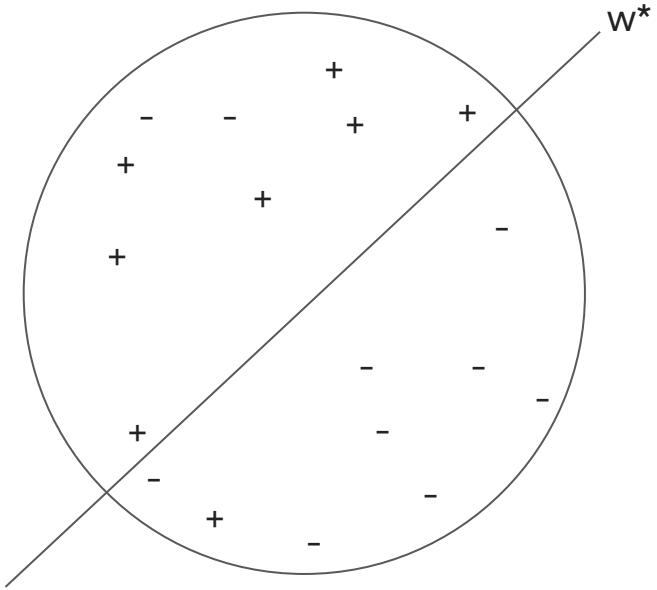

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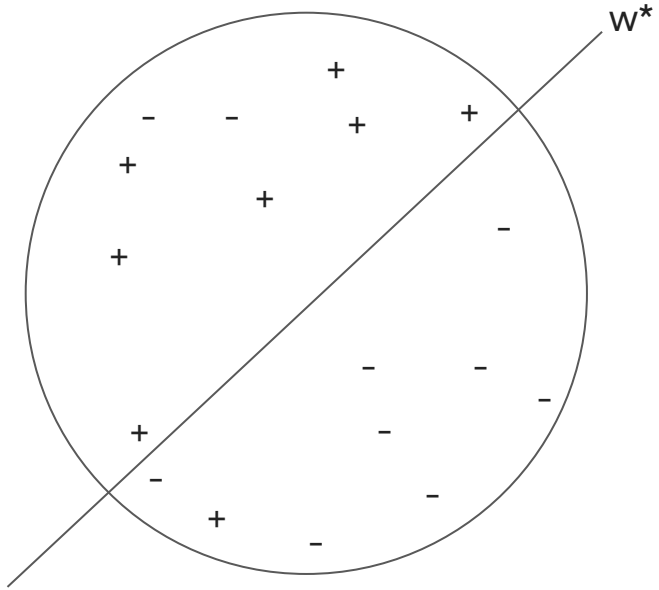

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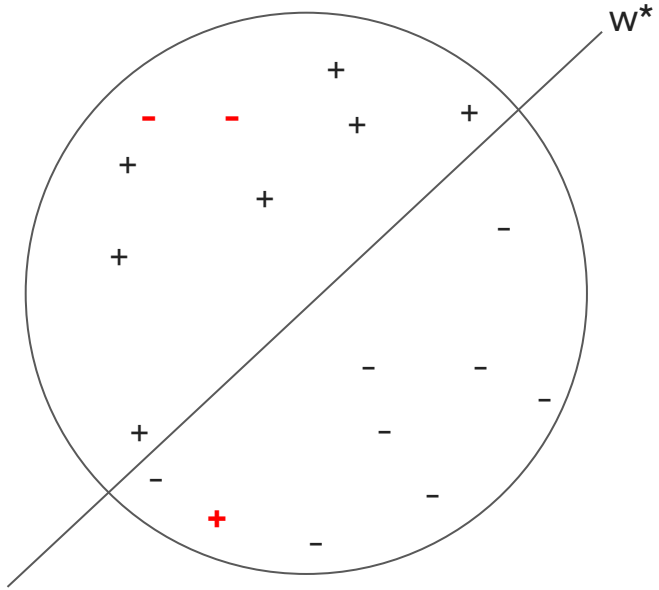

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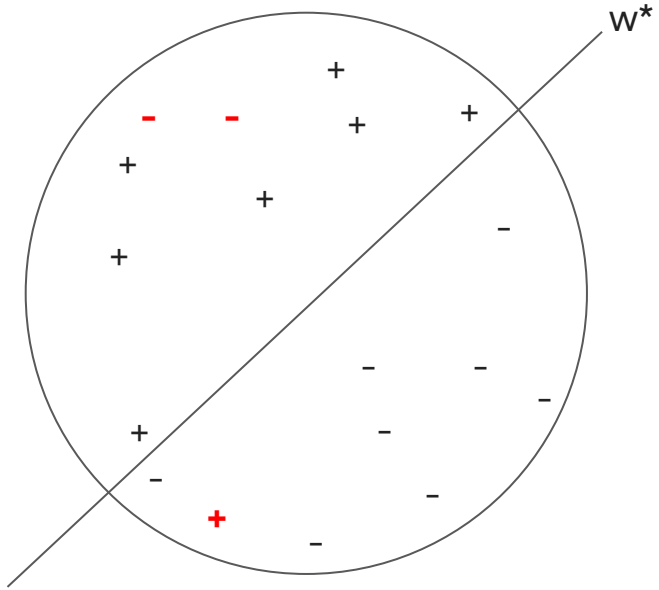

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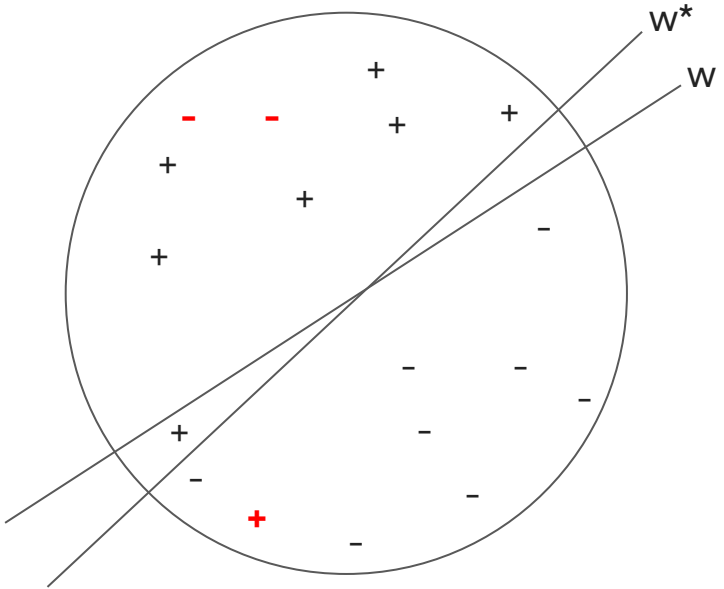

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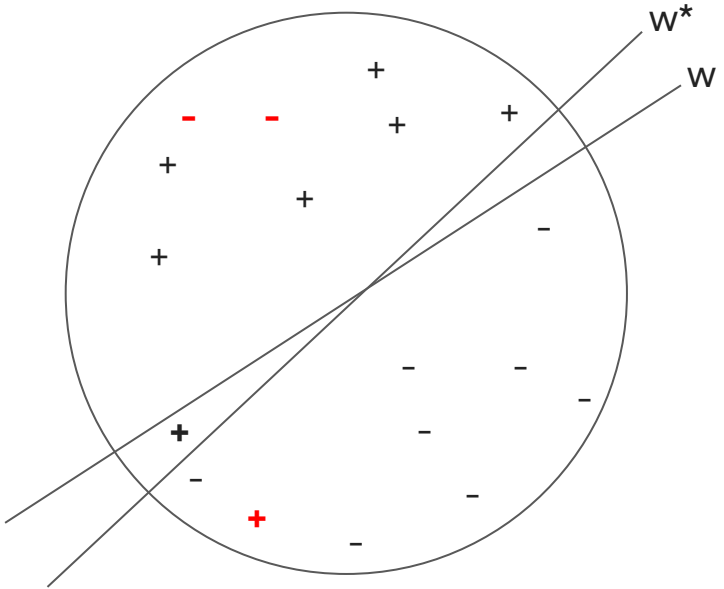

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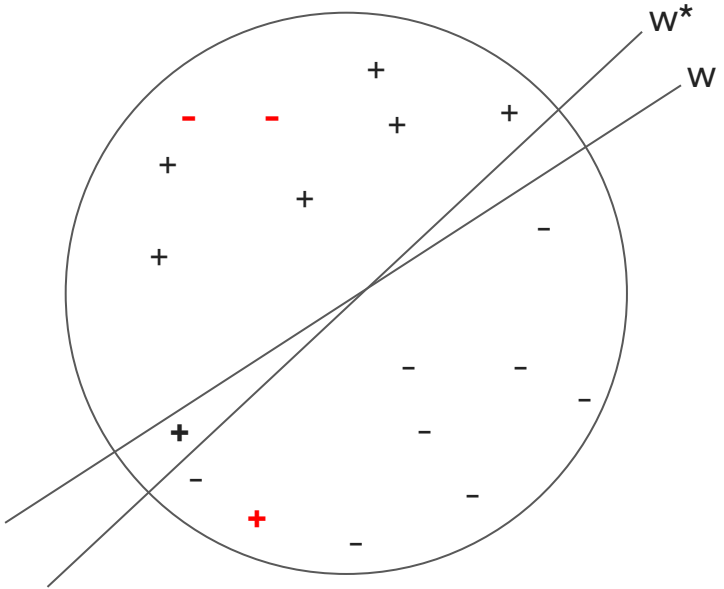

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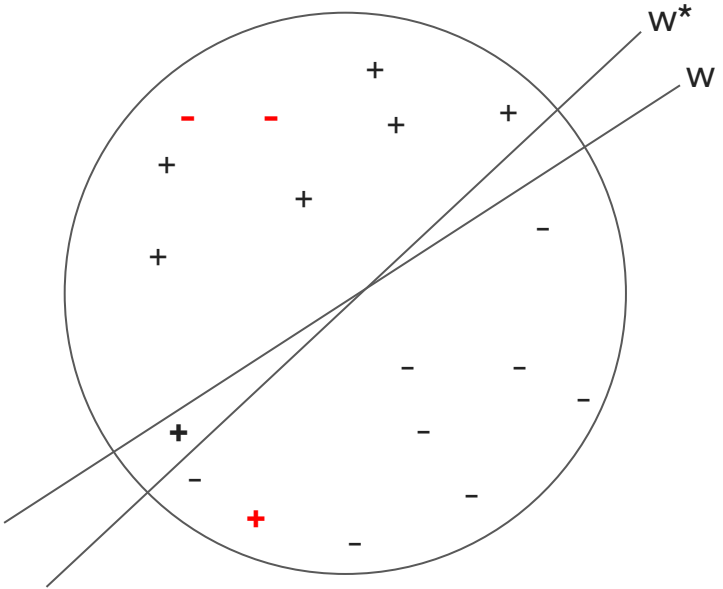
[Guruswami-Raghavendra' 06, Feldman et al.'06] Even weak learning is NP-hard.



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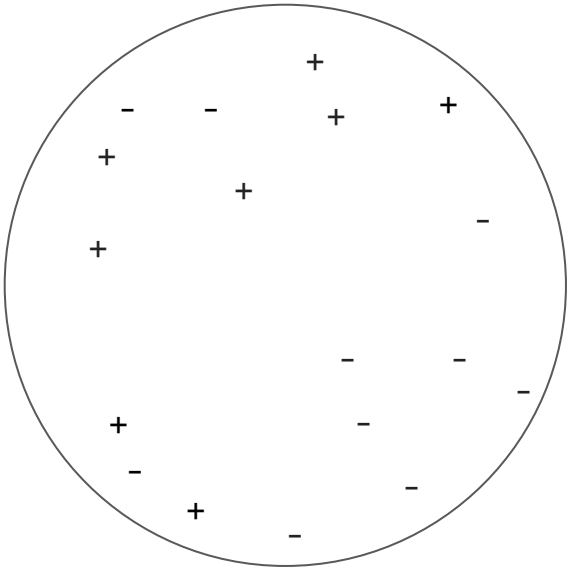

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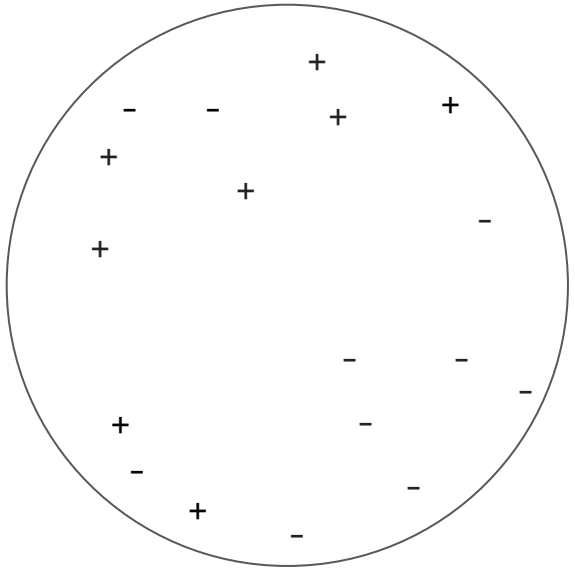

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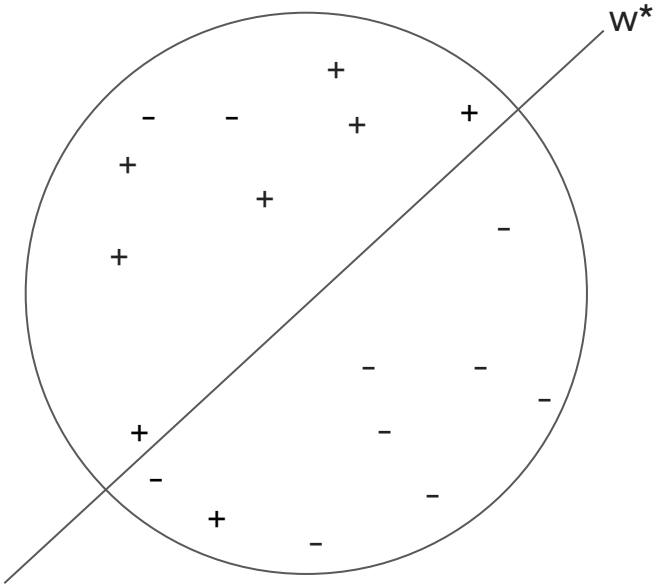

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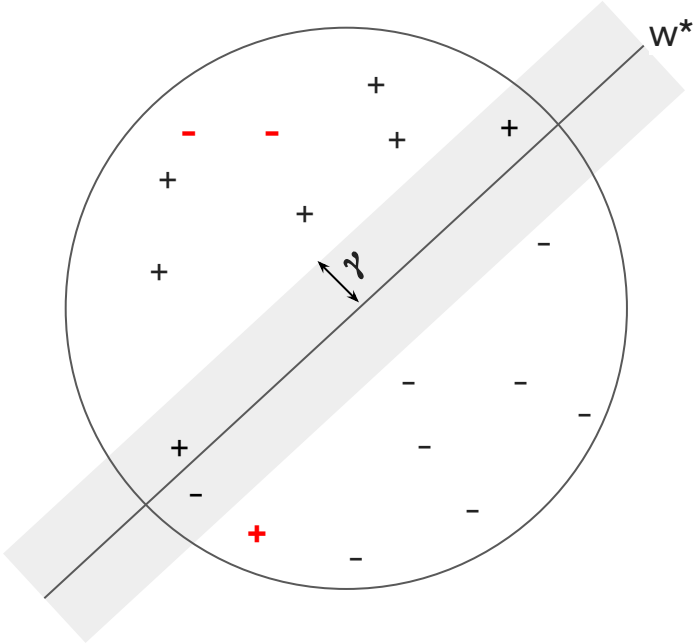

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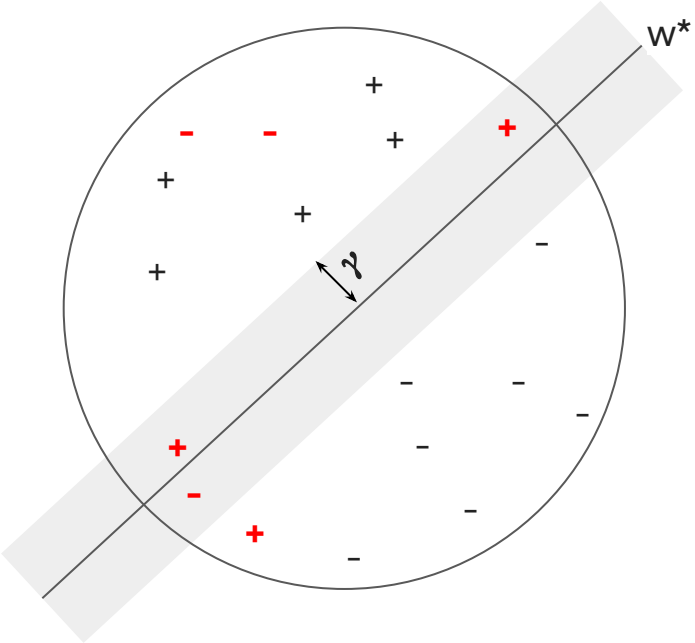

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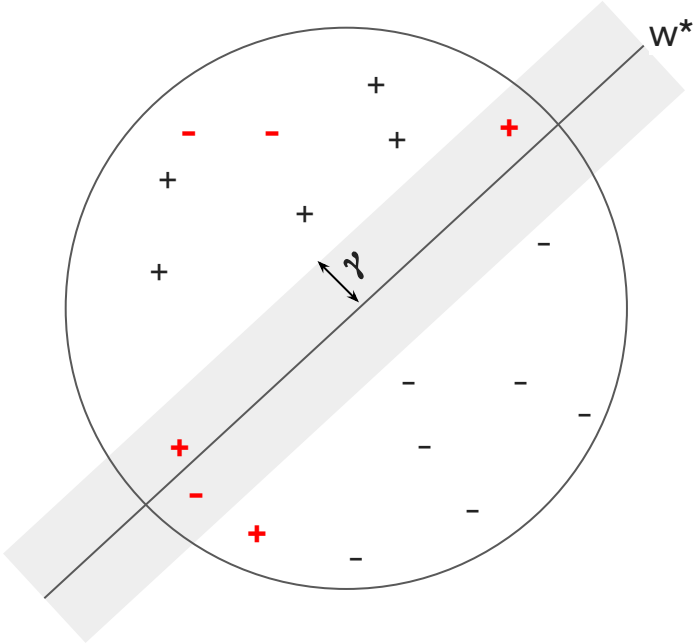

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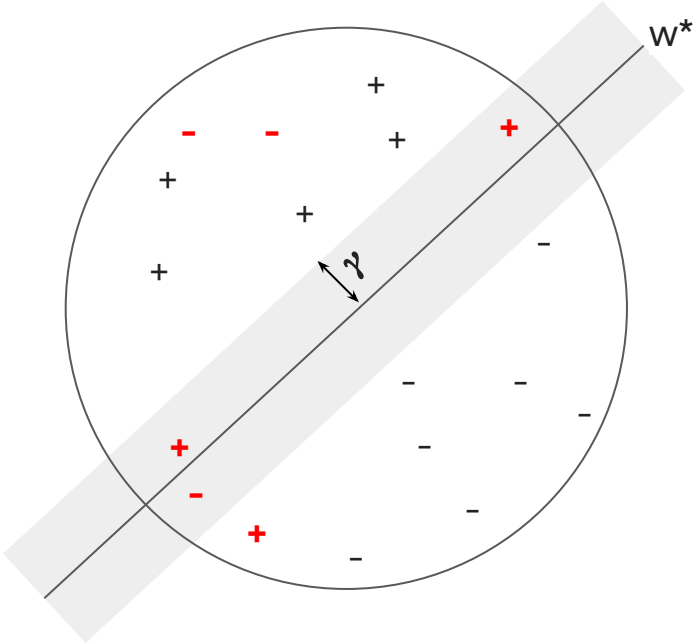

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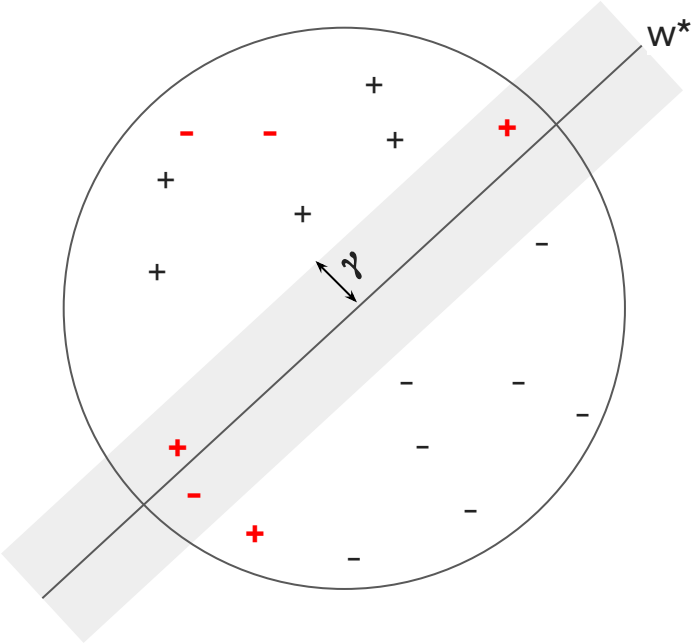
Margin Assumption

- "Robustness" of the optimal halfspace to ℓ_2 noise



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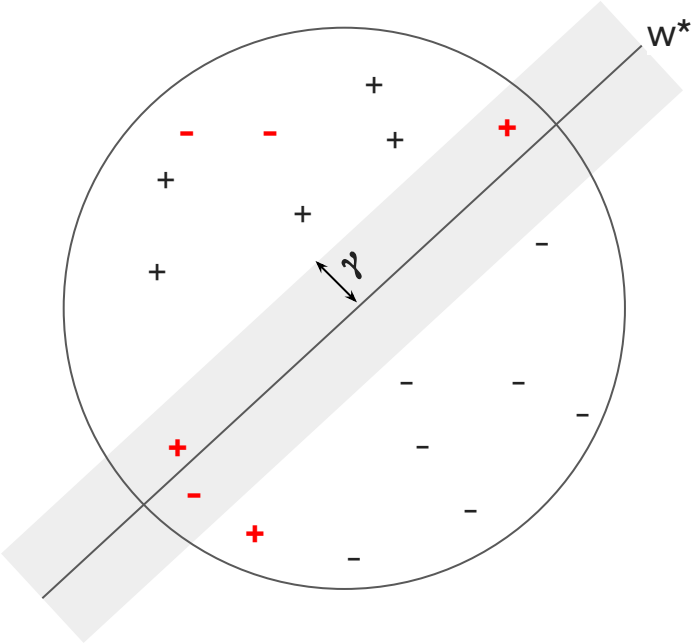
- Variants used in Perceptron, SVMs



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Agnostic Proper Learning of Halfspaces *with a Margin*

Previous Works

Our Results

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Previous Works

[Ben-David & Simon'00]
proper 1-learner that runs in $\mathrm{poly}(d) \cdot \exp(\tilde{O}(\log(1/\varepsilon)/\gamma^2))$ time, takes $O(1/\varepsilon^2\gamma^2)$ samples

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Theorem 2 Assuming *Exponential Time Hypothesis*, for any constant $\alpha > 1$, no *proper* α-learner runs in $\mathrm{poly}(d/\varepsilon) \cdot \exp(O(1/\gamma^{2-o(1)}))$ time

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runs in $\mathrm{poly}(d/\varepsilon) \cdot \exp(\tilde{O}(1/\gamma))$ time
takes $\mathrm{poly}(d/\varepsilon) \cdot \exp(\tilde{O}(1/\gamma))$ samples
Output hypothesis is not a halfspace

Our Results

Approximation ratio: any $\alpha > 1$

Theorem 1 *proper* 1.01-learner that runs in $\mathrm{poly}(d/\varepsilon) \cdot \exp(\tilde{O}(1/\gamma^2))$-time takes $O(1/\varepsilon^2\gamma^2)$ samples

Theorem 2 Assuming *Exponential Time Hypothesis*, for any constant $\alpha > 1$, no *proper* α-learner runs in $\mathrm{poly}(d/\varepsilon) \cdot \exp(O(1/\gamma^{2-o(1)}))$ time

Agnostic Proper Learning of Halfspaces *with a Margin*

Previous Works

Approximation ratio: $\alpha = 1$

[Ben-David & Simon'00]
proper 1-learner that runs in $\mathrm{poly}(d) \cdot \exp(\tilde{O}(\log(1/\varepsilon)/\gamma^2))$ time, takes $O(1/\varepsilon^2\gamma^2)$ samples

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Also results for large approximation ratio α

Main Algorithmic Idea

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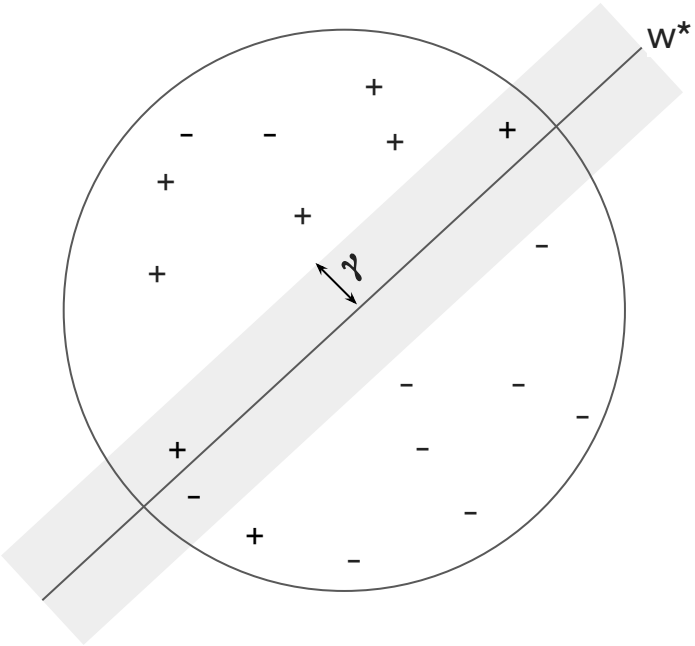

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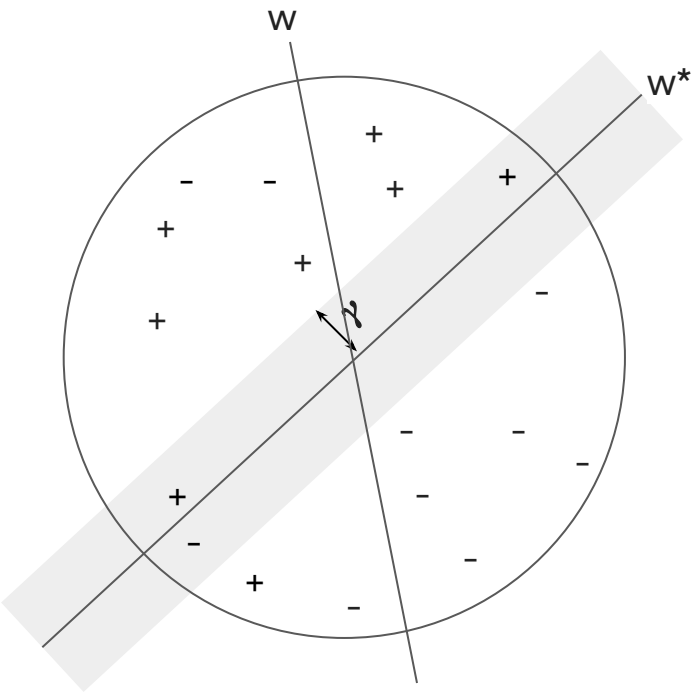

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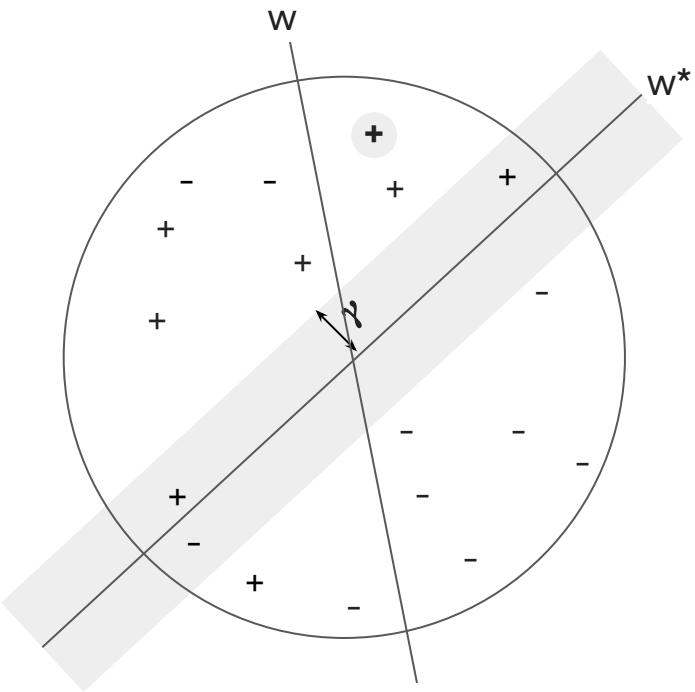

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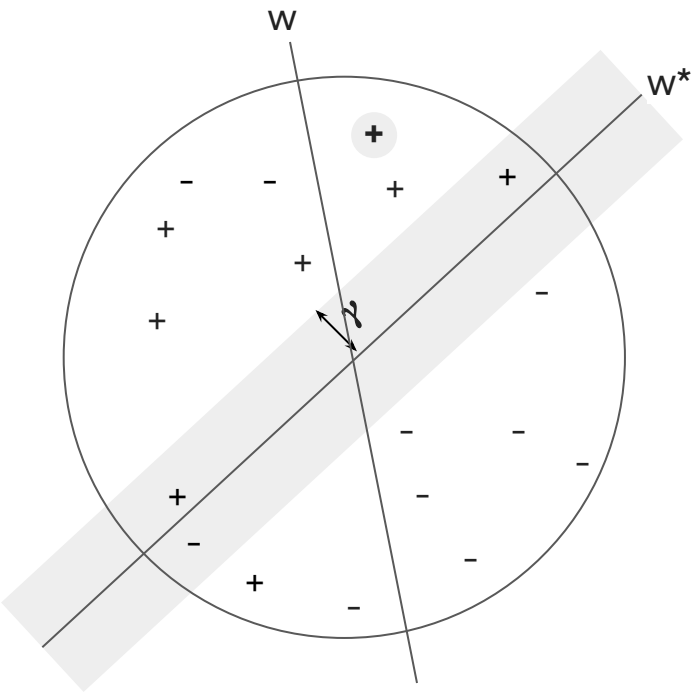
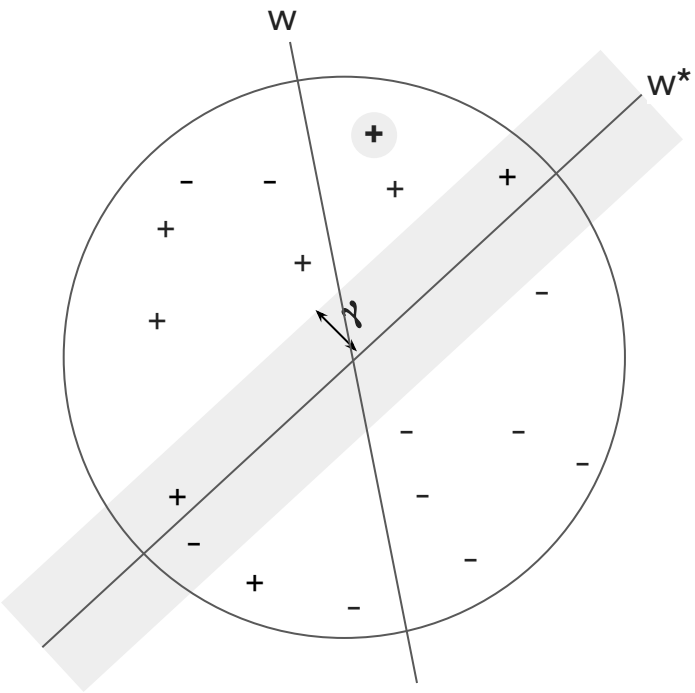


Observation Let (x, y) be a sample correctly classified by w* with margin γ, but incorrectly classified by w. Then, $\langle w^* - w, x \rangle \cdot y > \gamma$.

Main Algorithmic Idea

Rough Algorithm Outline

- Start with w = 0



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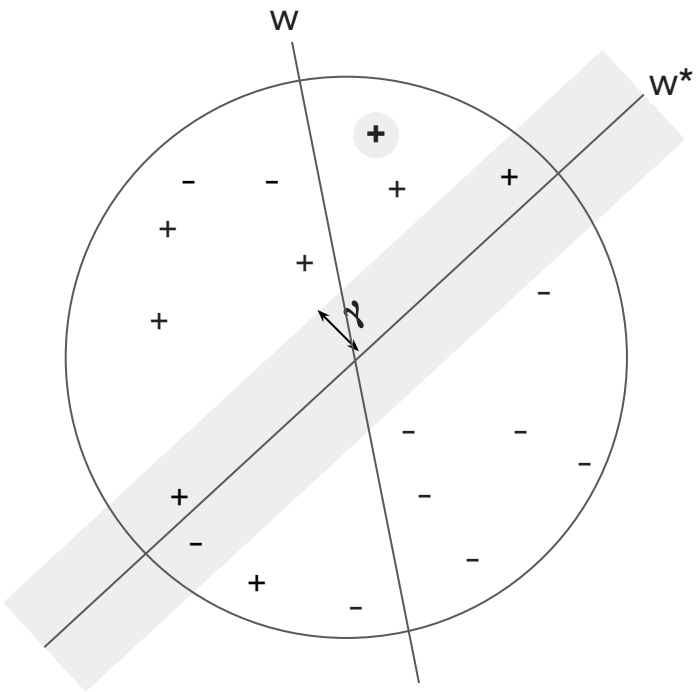

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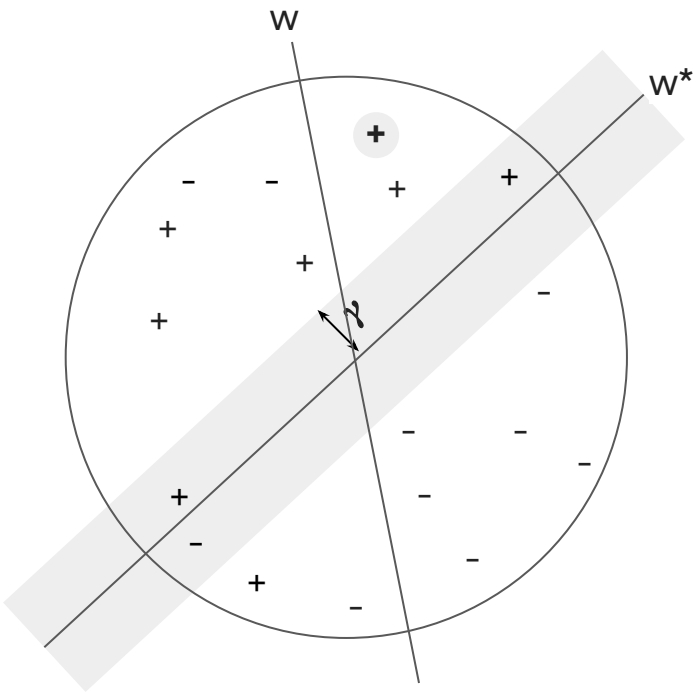

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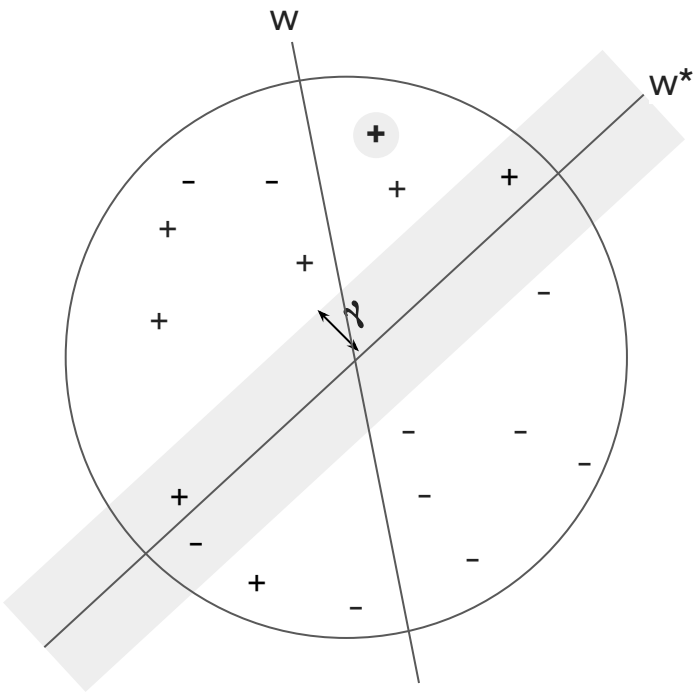

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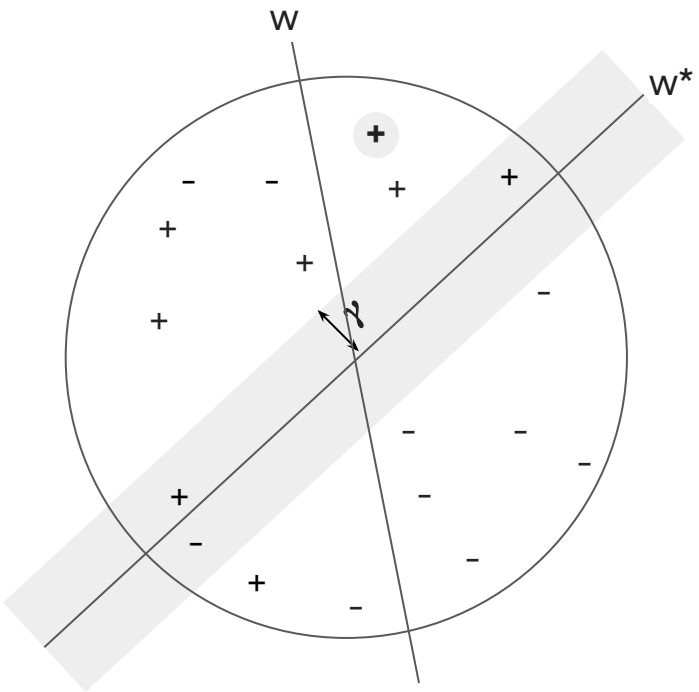

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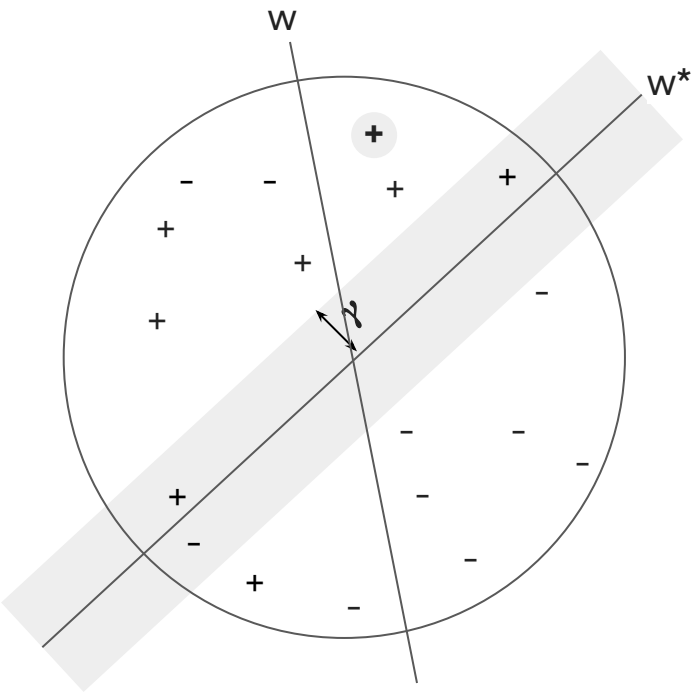

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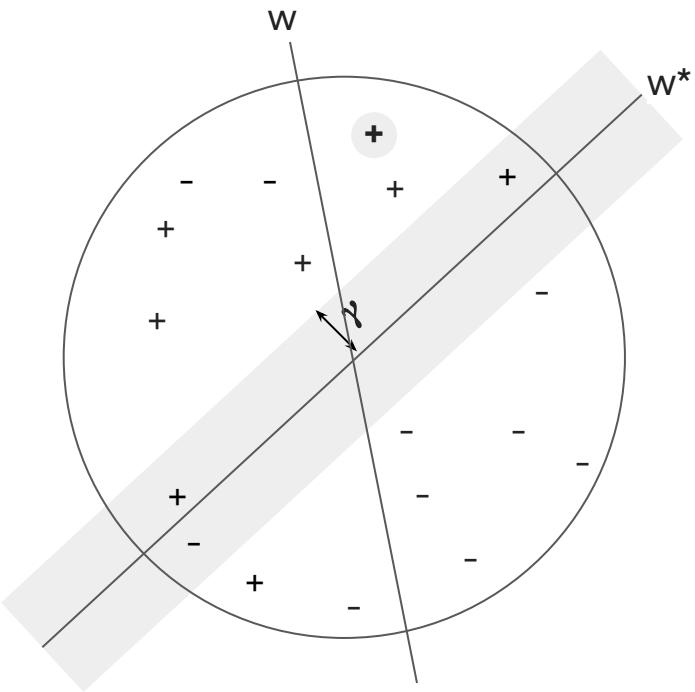

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